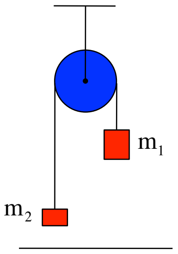
Problem 8.7

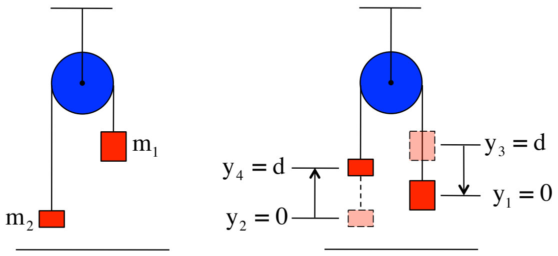
This system is called an *Atwood machine.*



Minor preliminary note: I have positioned the floor so that m_2 is not initially sitting on it. I'm doing this because having m_2 starting out on the floor obscures an important theoretical twist of which you should be aware.

Specifically, when dealing with *gravitational potential energy* close to the surface of the earth, you can define a body's *zero-potential-energy level* to be *anywhere you want* (this is because there is no "preferred" place near the earth's surface where the gravitational force is itself zero). That means you can assign a "y = 0" level FOR EACH BODY IN THE SYSTEM. Indeed, it is sometimes convenient to assign the same coordinate axis for all of the bodies in a system, but it doesn't *have* to be done that way. In fact, with multiple body systems like those found in the Atwood machine, I usually define "y = 0" to be at each body's lowest point. That is what I have done in this problem. You can see the set-up on the next page.

The set-up:



With the set-up as shown, m_2 starts at $y_2 = 0$ and rises to $y_4 = d$. The mass m_1 starts at $y_3 = d$ and drops to $y_1 = 0$. With that, the *conservation of energy* follows.

NOTES:

--The work tension does on m_1 is equal and opposite the work tension does on m_2. As such, tension isn't usually included in this calculation. For here, I will include it just to be absolutely complete. You shouldn't on a test.

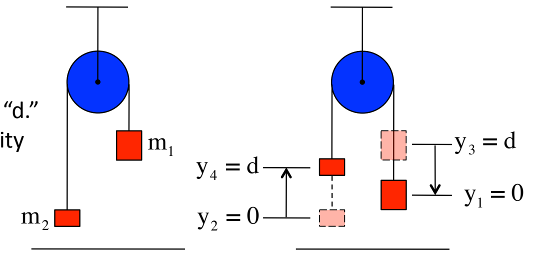
--With no net work being done by tension, and no friction in the system, there is no net extraneous work being done in the system. That means that normally, if I wasn't including tension, the $\sum W_{ext} = 0$ would be zero.

NOTES (cont'd):

--The system begins from rest.

--Both masses move a distance "d."

--Both masses have same velocity magnitude at the end of the interval.



$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + m_1 g(d) + \left(W_{T\_on\_m_1} + W_{T\_on\_m_2}\right) = \left(\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2\right) + m_2 g(d)$$

$$0 + m_1 g(d) + \left(\cancelto{0}{Td\cos 180^\circ + Td\cos 0^\circ}\right) = \left(\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2\right) + m_2 g(d)$$

$$\Rightarrow \quad v = \left[\frac{2\left[m_1 g(d) - m_2 g(d)\right]}{(m_1 + m_2)}\right]^{1/2}$$

$$\Rightarrow \quad v = \left[\frac{2\left[(5.00\text{ kg})(9.80\text{ m/s}^2)(4.00\text{ m}) - (3.00\text{ kg})(9.80\text{ m/s}^2)(4.00\text{ m})\right]}{(5.00\text{ kg}) + (3.00\text{ kg})}\right]^{1/2}$$

$$= 4.43\text{ m/s}$$

b.) Once m_2 gets to the end of the interval, it has speed $v_1 = 4.43\text{ m/s}$ and it begins to freefall (it does this because in the original problem, the other mass stopped after hitting ground). Once freefalling, we can use *Conservation of Energy* to determine it's additional distance traveled. That is:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$\frac{1}{2}m_2 {v_1}^2 + 0 + 0 = 0 + m_2 gh$$

$$\Rightarrow \quad h = \frac{\left(\frac{1}{2}v^2\right)}{g}$$

$$= \frac{.5(4.43\text{ m/s})^2}{(9.80\text{ m/s}^2)}$$

$$= 1.00\text{ m} \quad \Rightarrow \quad \text{so the height from the tabletop is 5.00 m}$$

Note that using kinematics also works. That is:

$$(v_{y,2})^2 = (v_{y,1})^2 + 2a_y \Delta y \quad \Rightarrow \quad 0 = (4.43\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)h \quad \Rightarrow \quad h = 1.00\text{ m, etc.}$$